Image Segmentation and Classification
Lecture 5

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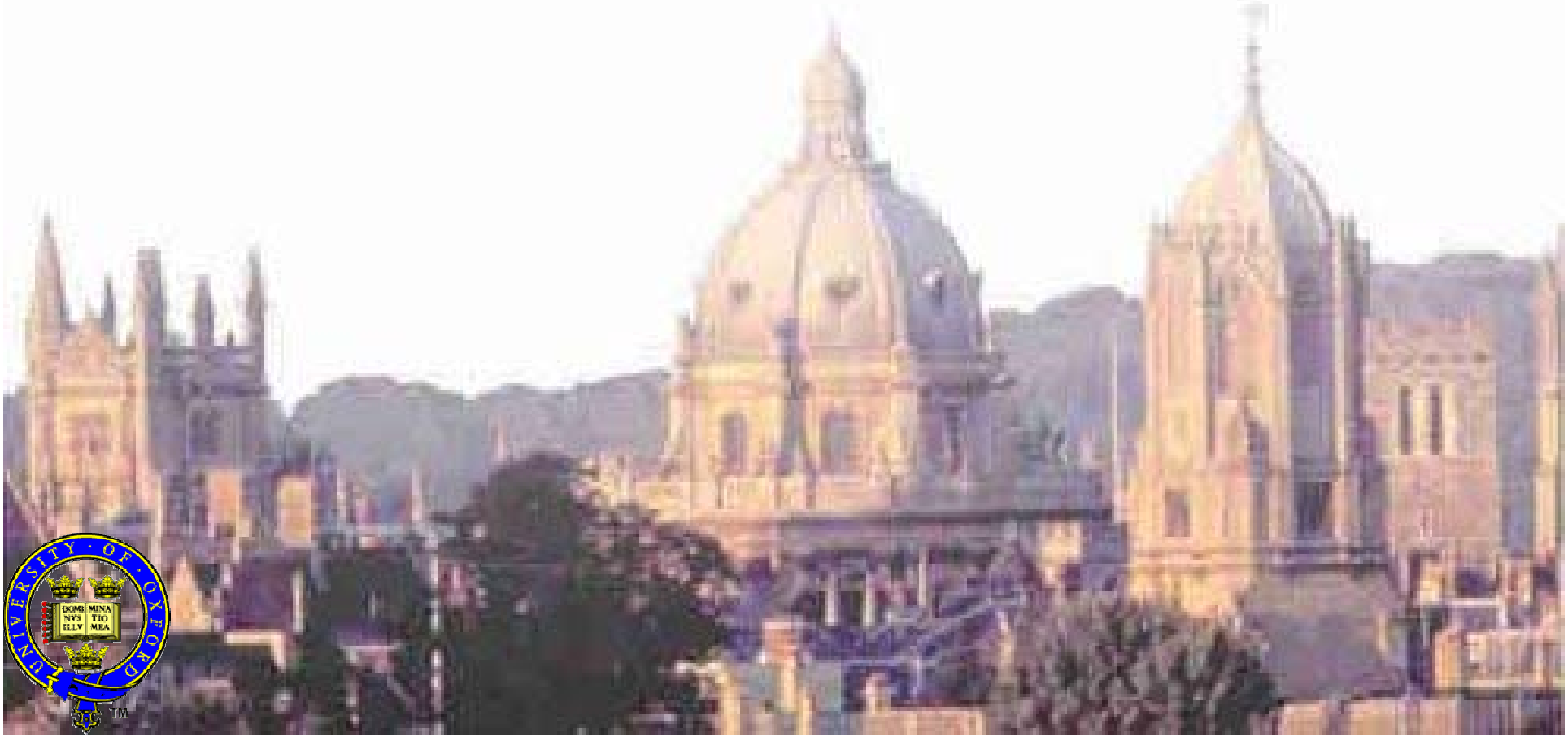
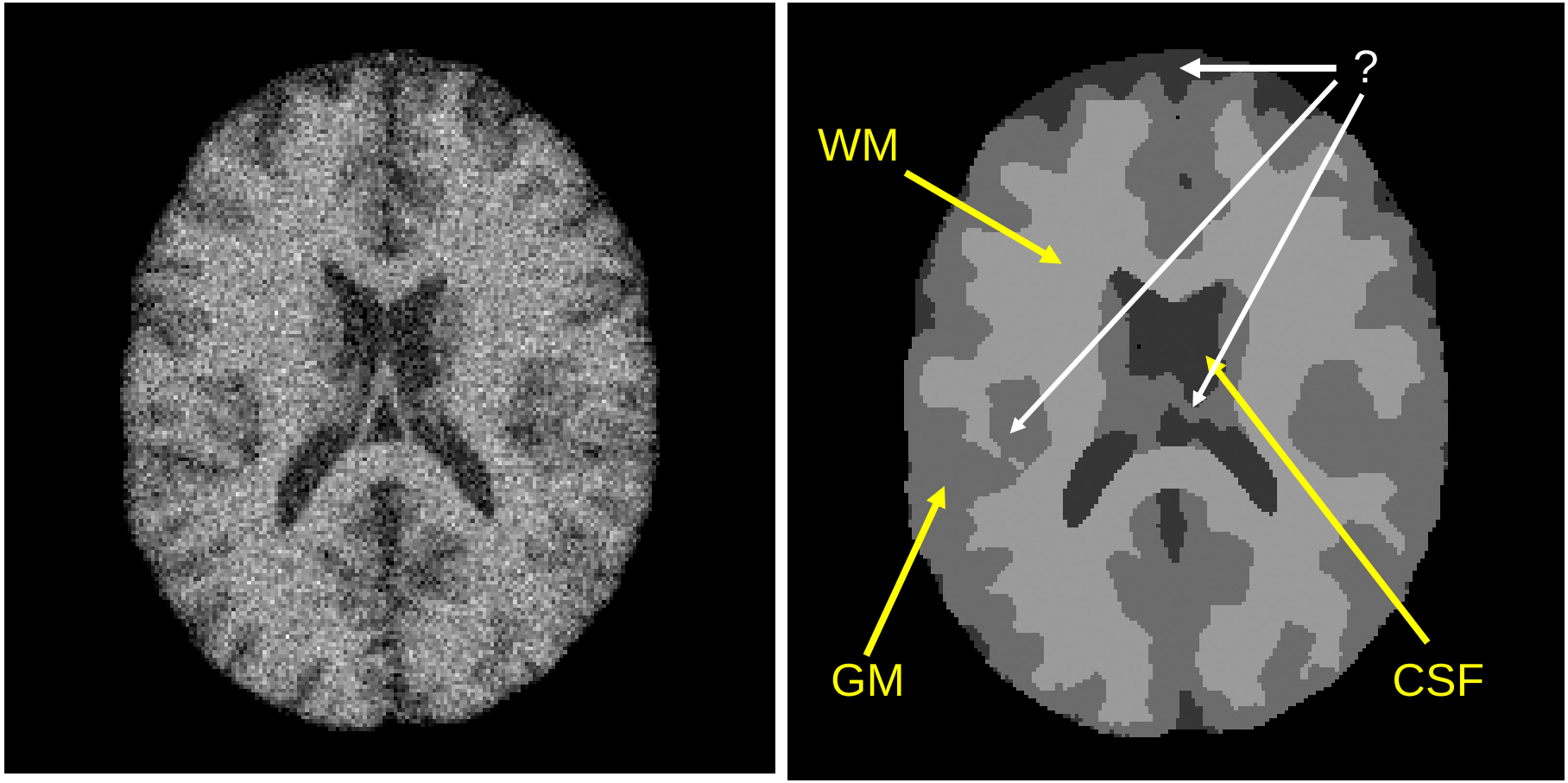
Hilary Term 2005



The goal

- Segmentation means to divide up the image into a patchwork of regions, each of which is “homogeneous”, that is, the “same” in some sense
  - Intensity, texture, colour, …
- Classification means to assign to each point in the image a tissue class, where the classes are agreed in advance
  - Grey matter (GM), White matter (WM), cerebrospinal fluid (CSF), air, … in the case of the brain
- Note that the problems are inter-linked: a classifier implicitly segments an image, and a segmentation implies a classification

MRI brain slices



The noisy MRI image of the brain slice shown left is ideally piecewise constant, comprising grey matter, white matter, air, ventricles. The right image is a segmentation of the image at left. Evidently, while it is generally ok, there are several errors. Brain MRI is as easy as it gets!!

Medical image segmentation is generally difficult

- Noisy images
  - often Noise-to-signal is 10%
  - This is ten times N/S of camera images
- Often textured in complex ways
- Relatively poorly sampled
  - Many pixels contain more than one tissue type … this is called Partial Volume Effect
- Objects of interest have complex shapes
- Signs of clinical interest are subtle

Even MRI image segmentation is hard

+ Excellent contrast between soft tissues
+ Brain images are approximately piecewise constant; but complex textures in other organs
  - Classification ought to be easy: GM, WM, CSF, air
- There are image distortions (eg motion artefact, bias field, …)
- Partial volume effect (PVE)
- structures of interest (tumours, temporal lobes, …) have complex shapes

Classification of brain MRI images

- The “labels” we wish to assign to objects are typically few and known in advance
  - e.g. WM, GM, CSF and air for brain MRI
- objects of interest usually form coherent continuous shapes
  - If a pixel has label c, then its neighbours are also likely to have label c
  - Boundaries between regions labelled c, d are continuous
- Image noise means that the label to be assigned initially at any pixel is probabilistic, not certain

One way to accommodate these considerations is Hidden Markov Random Fields

Segmentation by classification of voxels

Every pixel is classified according to its probability of being a member of a particular tissue class.

The Maximum Likelihood (ML) estimator assigns to each voxel **x** that class which maximises

$$\Pr(\mathbf{x} \in C), \text{ where } C \in \{WM, GM, CSF, \ldots\}$$

The Maximum a Posteriori (MAP) estimator

$$P(\mathbf{x} \mid \mathbf{y}) = P(\mathbf{y} \mid \mathbf{x}) P(\mathbf{x}) / P(\mathbf{y})$$

Normalising intensities: $P_{MAP}(\mathbf{x} \mid \mathbf{y}) = P(\mathbf{y} \mid \mathbf{x}) P(\mathbf{x})$

In practice, neither ML nor MAP work well on brain MRI.

To understand why, we need to model the probability of a pixel, with a particular intensity, being a member of a particular tissue class such as GM, WM, …

Each class is defined as a probability density function

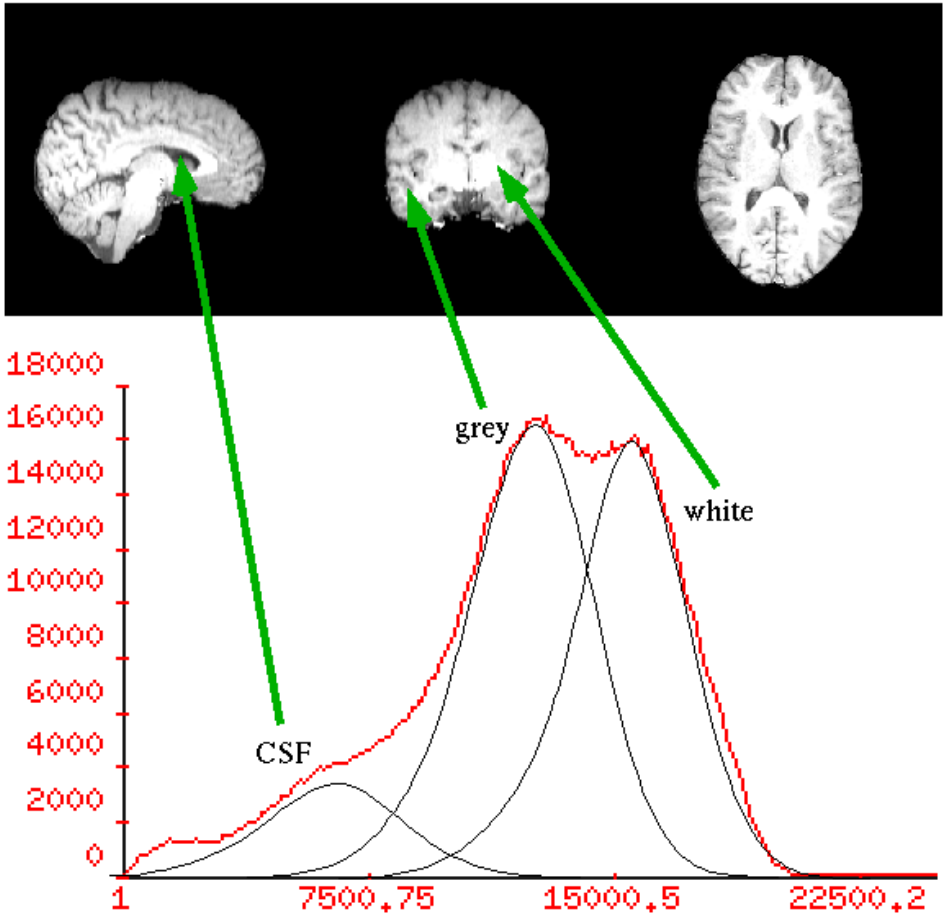
Each class, say the one with label l (= GM, say) has an associated PDF, with parameters θ_l

Often θ_l is a Gaussian $\theta_l = (\mu_l, \sigma_l)$

$$p(y_i \mid l) = f(y_i; \theta_l)$$

$$= \frac{1}{\sqrt{2\pi\sigma_l^2}} \exp\left(- \frac{(y_i - \mu_l)^2}{2\sigma_l^2} \right)$$

Tissue Probability Density Functions



Note the huge overlap between the Gaussian pdf for GM and that for WM

This means that there are many misclassifications of GM pixels as WM and vice versa.

Even small amounts of noise can change the ML classification.

Can we do better?

(a) take spatial information into account; and

(b) model expected spatial image distortions.

We address both issues using a MRF.

MRF model

A lattice of sites = the pixels of an image $S = \{1,..., N\}$

A family of random variables, whose values define a configuration

$$R = \{r_i, i \in S\}$$

$$(R_1 = r_1,..., R_N = r_N) \equiv R = r$$

The set of possible intensity values $D = \{1,..., d\}$

An image viewed as a random variable $Y = \{\mathbf{y} = (y_1,..., y_N) \mid y_i \in D, i \in S\}$

The set of class labels for each pixel (eg GM, WM, CSF) $L = \{1,..., l\}$

A classification (segmentation) of the pixels in an image $X = \{\mathbf{x} = (x_1,..., x_N) \mid x_i \in L, i \in S\}$

Pairwise independence

Classification is independent of image *and* neighbouring voxels are independent of each other

$$P(\mathbf{y},\mathbf{x}) = \prod_{i \in S} P(y_i, x_i)$$

Assume a parametric (eg Gaussian) form for distribution of intensity given label l

$$p(y_i \mid l) = f(y_i; \theta_l)$$

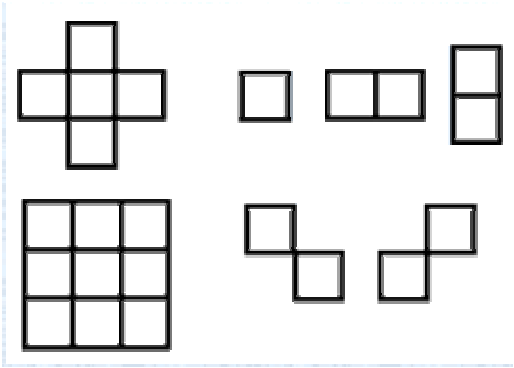
Markovian assumptions: probability of class label *i* depends only on the local neighbourhood N_i

$$P(\mathbf{x}) > 0, \forall \mathbf{x} \in X$$

$$P(x_i \mid x_{S-\{i\}}) = P(x_i \mid x_{N_i})$$

MRF equivalent to Gibbs distributions

- Deep theorem of Hammersley & Clifford
  - MRFs are equivalent to systems with a local potential function
- Define *cliques* made of neighbouring pixels



- Each clique has a local "potential energy" *U* which enables the probability of the pixel label to be calculated efficiently

$$\log P(\mathbf{x} \mid \mathbf{y}) = -U(\mathbf{x} \mid \mathbf{y})$$

$$\text{Equivalently}: P(\mathbf{x} \mid \mathbf{y}) = \exp(-U(\mathbf{x} \mid \mathbf{y}))$$

MRF-MAP estimation

We seek a labelling of the image according to the MAP criterion:

$$\hat{x} = \arg\max_{\mathbf{x} \in X} \left\{ P(\mathbf{y} \mid \mathbf{x}) P(\mathbf{x}) \right\}$$

Recall that

$$P(x) = \frac{1}{Z} \exp(-U(x))$$

And, if we have Gaussian distributions of pixel values for each class, and $x_i = l$

$$p(y_i \mid x_i) = \frac{1}{\sqrt{2\pi\sigma_l^2}} \exp\left(-\frac{(y_i - \mu_l)^2}{2\sigma_l^2} \right)$$

Pixel-wise independence & ICM

$$P(\mathbf{y} \mid \mathbf{x}) = \prod_{i \in S} p(y_i \mid x_i)$$

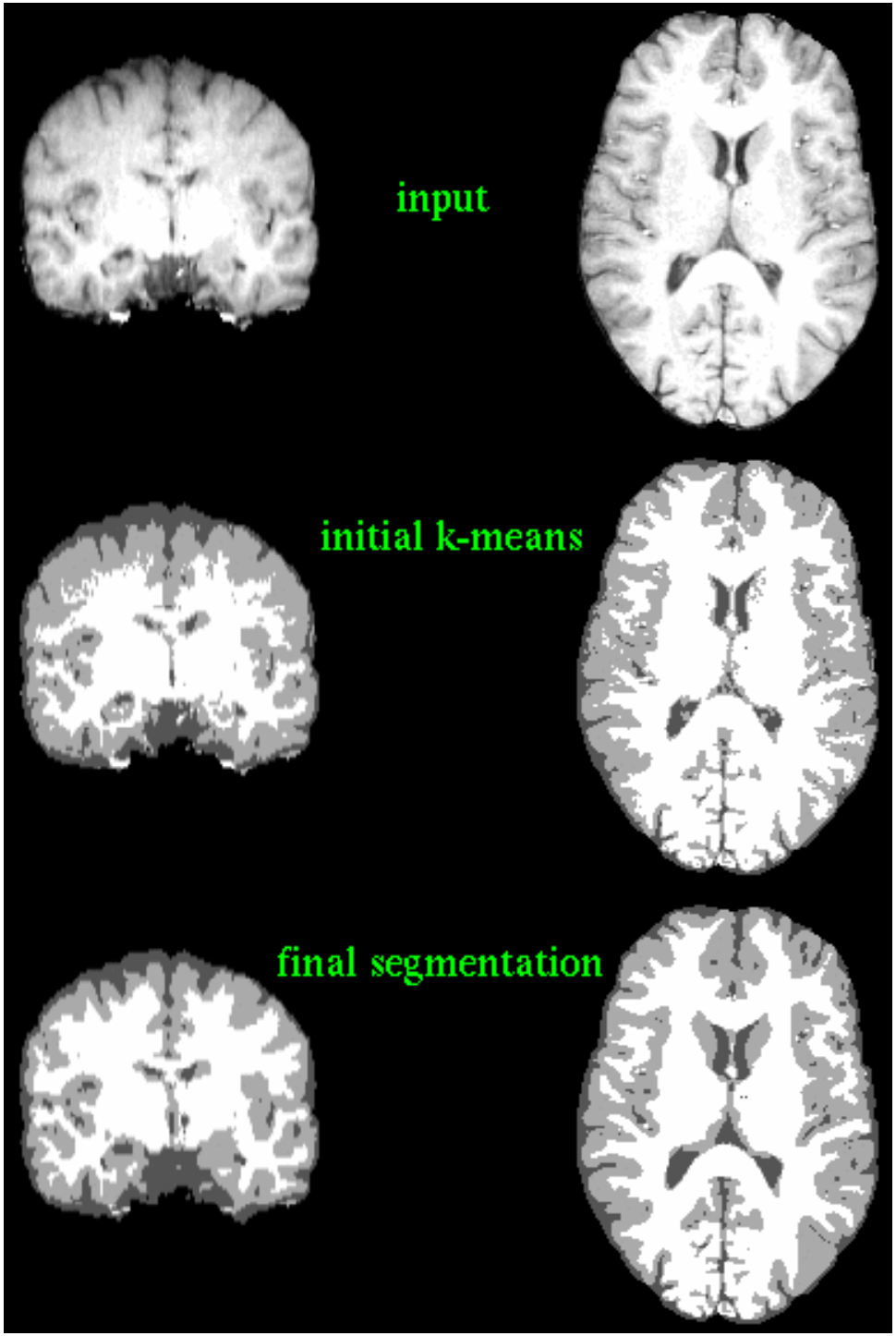
So that

$$U(\mathbf{y} \mid \mathbf{x}) = \sum_{i \in S} \left[\frac{(y_i - \mu_l)^2}{\sigma_l^2} + \log \sigma_l \right]$$

The final step is to use Besag's Iterated Conditional Modes algorithm:

$$\text{Update the class label } x_i^k \text{ at iteration } k$$

$$\text{by minimising } U(x_i \mid \mathbf{y}, x_{N_i - \{i\}})$$


input
initial k-means
final segmentation

Model estimation HMRF-EM

We have assumed that we know the model parameters $\theta_l = (\mu_l, \sigma_l)$ ahead of time. Unfortunately, this is rarely the case. The next idea is to estimate the parameters and do the segmentation cooperatively using the EM algorithm

Start : make initial estimates $\theta_i, i = 1...$

E - step : calculate conditional expectation

$$Q(\theta \mid \theta^k) = \sum_{x \in X} p(\mathbf{x} \mid \mathbf{y}, \theta^k) \log(p(x, y \mid \theta)$$

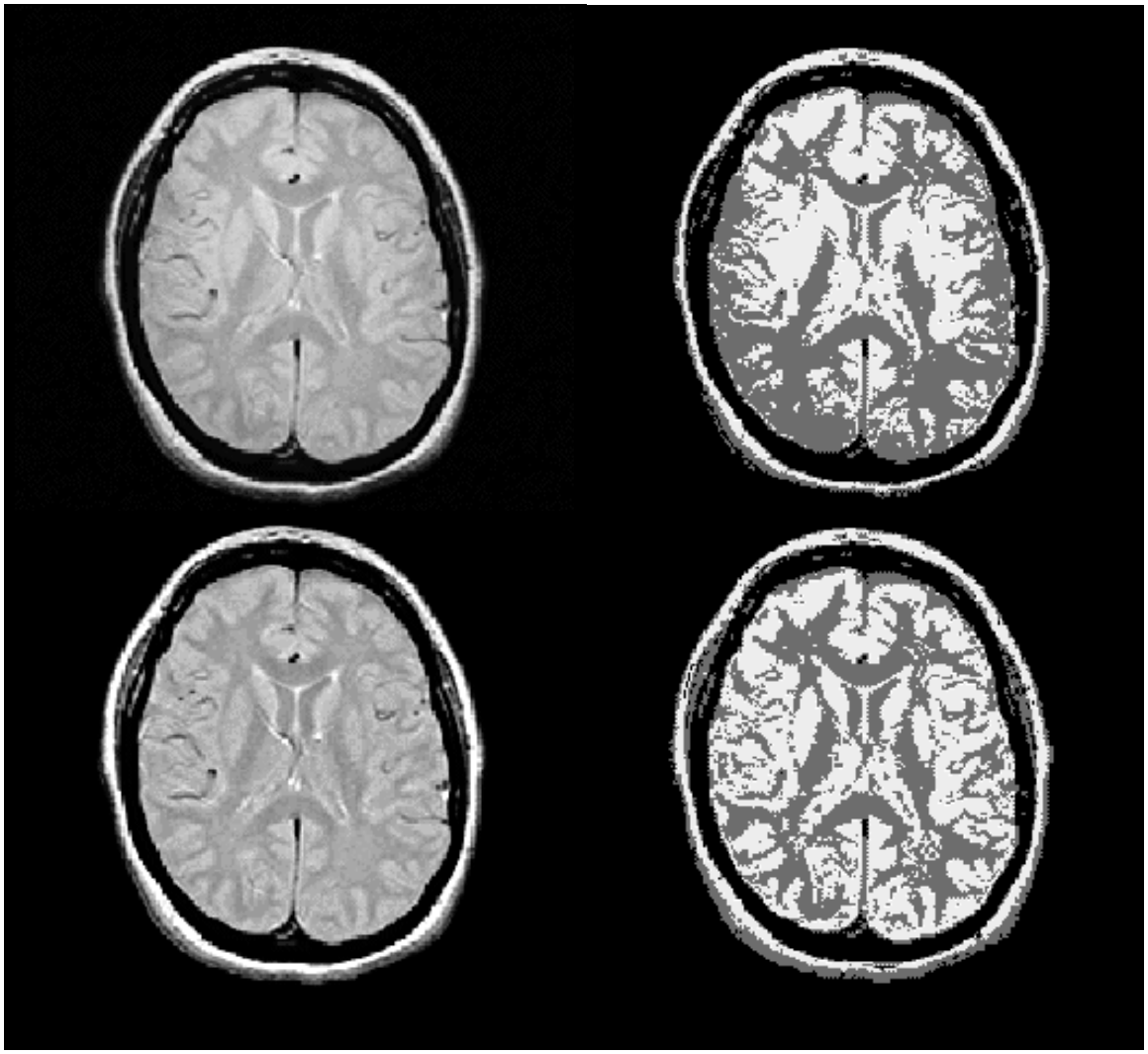
M - step : update estimate of class parameters

$$\theta^{k+1} = \arg\max_{\theta} Q(\theta \mid \theta^k)$$

Need to correct image distortion

Original image

Corrected image

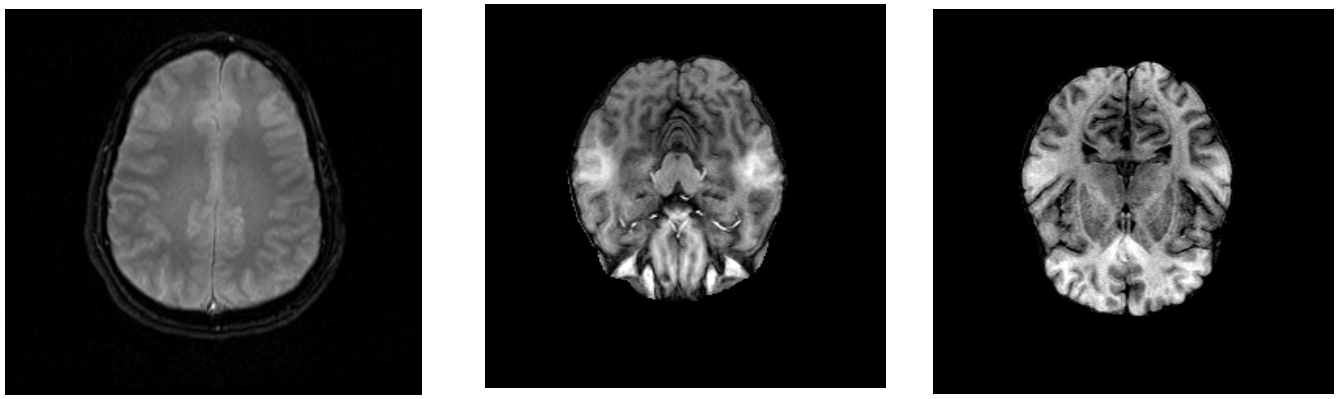


threshold to find white matter

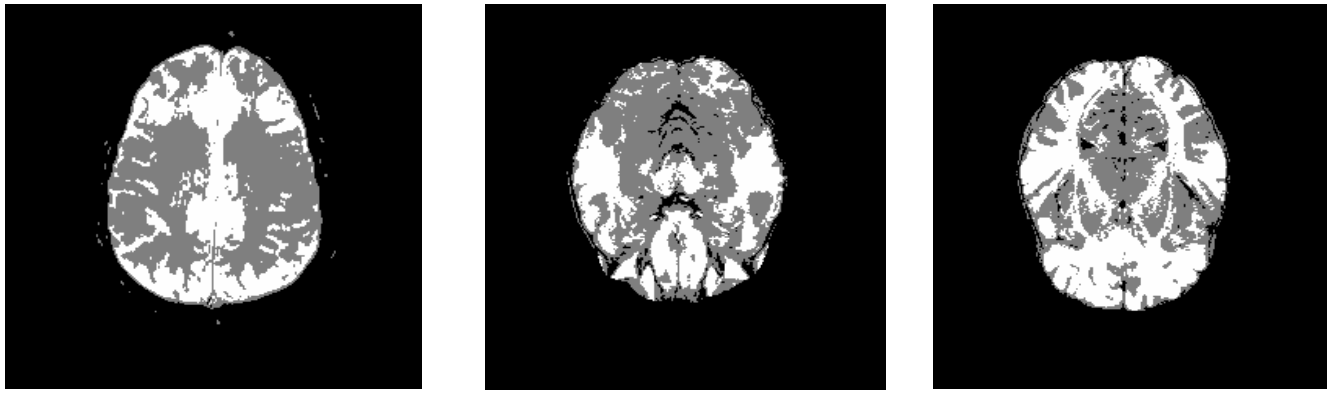
threshold to find white matter

Classification/segmentation without bias correction

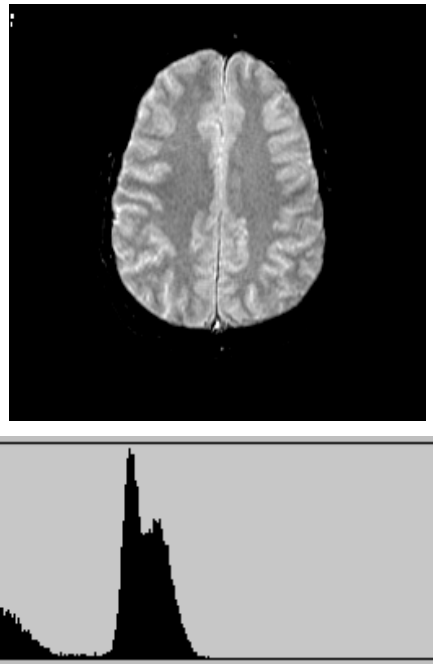
Typical MR images



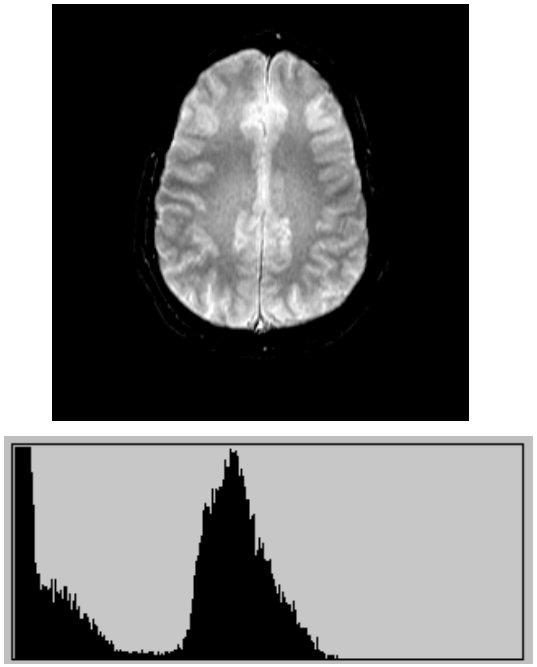
Segmentation misses lots of grey matter



Bias field affects image histogram (pdf)



Original image without bias field and its histogram



Bias field corrupted image and its histogram

Estimating the bias field amounts to estimating the pdf, given prior assumptions about the bias field and expected pdf

Applying HMRF to bias correction

E step

Compute MAP estimates for the bias field and classification

$$B^{(t)} = \arg\max_{B} p(B \mid \mathbf{y}, \mathbf{x}^{(t-1)}, \theta^{t})$$

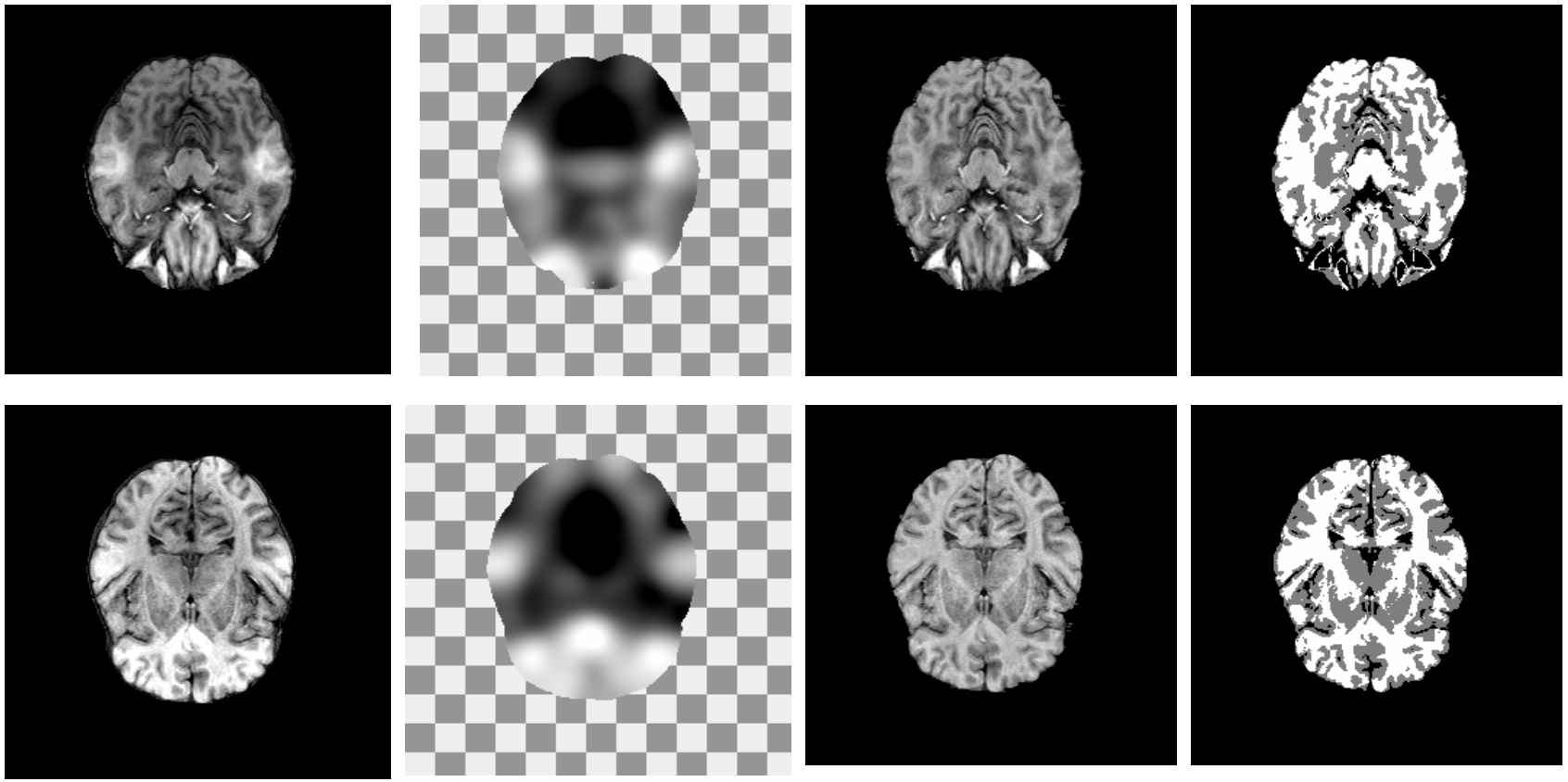
$$\mathbf{x}^{t} = \arg\min_{\mathbf{x} \in X} U(\mathbf{x} \mid \mathbf{y}, B^{t}, \theta^{t})$$

M step

Compute ML estimates of the parameters of the classes given current information

$$\theta^{(t+1)} = \arg\max_{\theta} P(\mathbf{y} \mid \theta, \mathbf{x}^{t}, B^{t})$$

Bias field correction



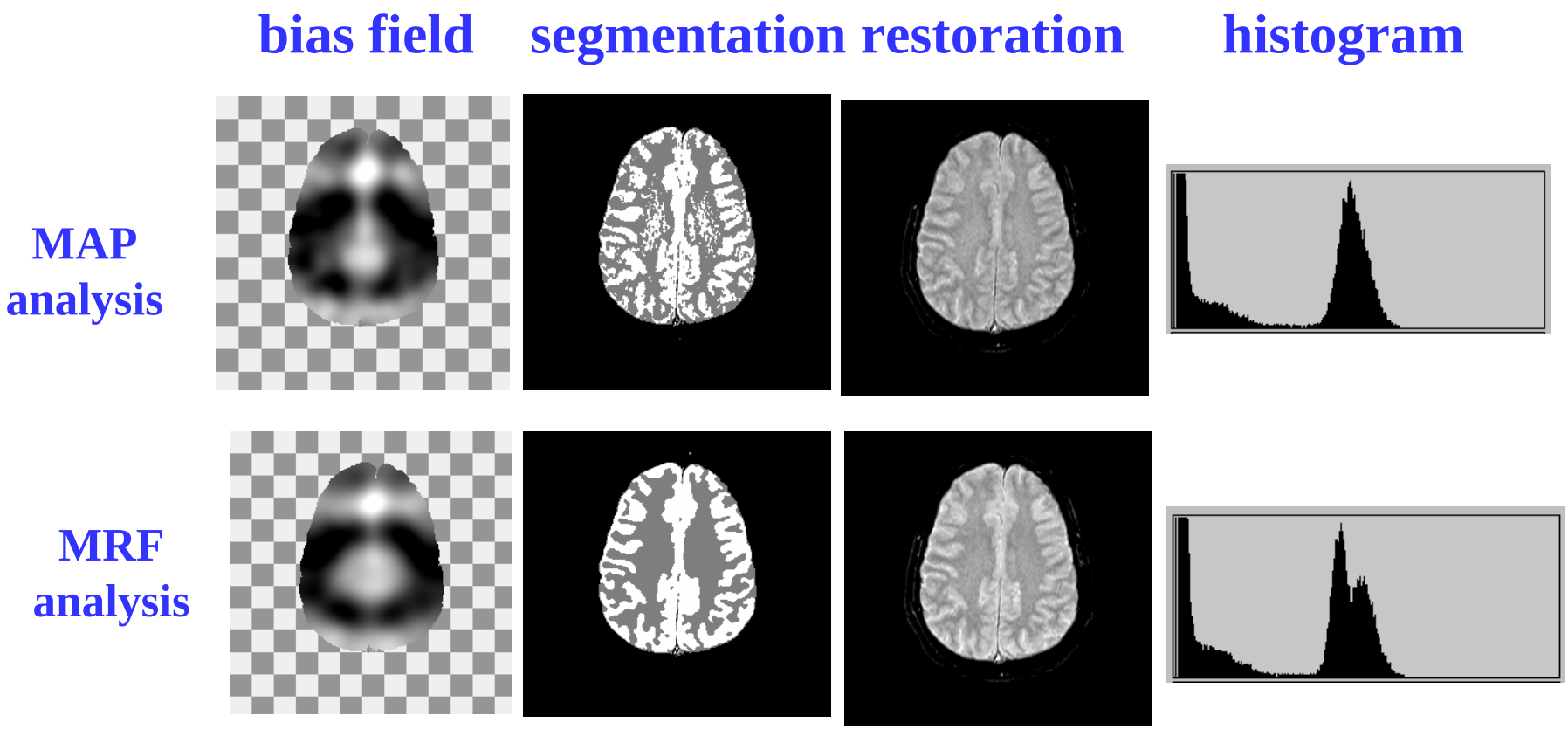
Original image

Estimated bias field

Corrected image

Improved segmentation

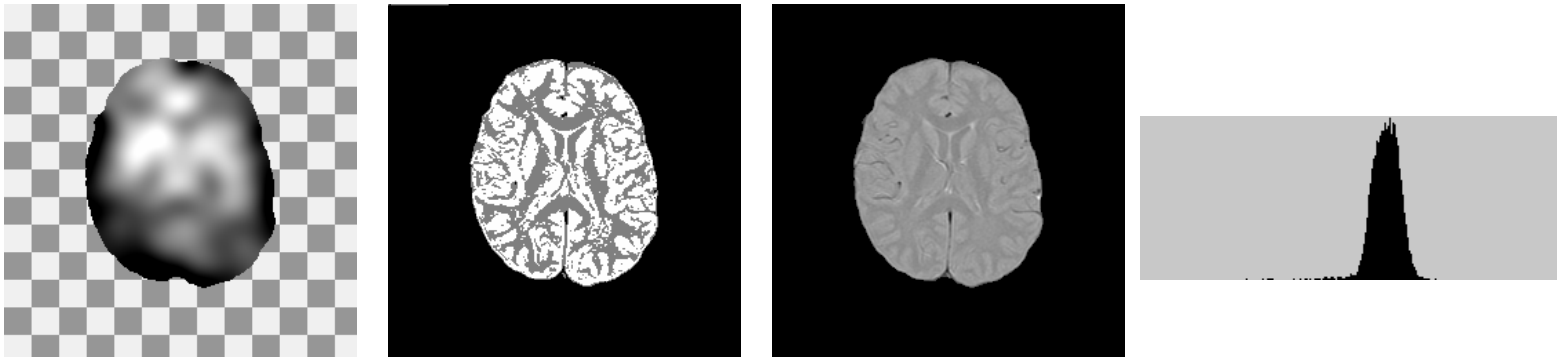
Experiments on Brain MR Images



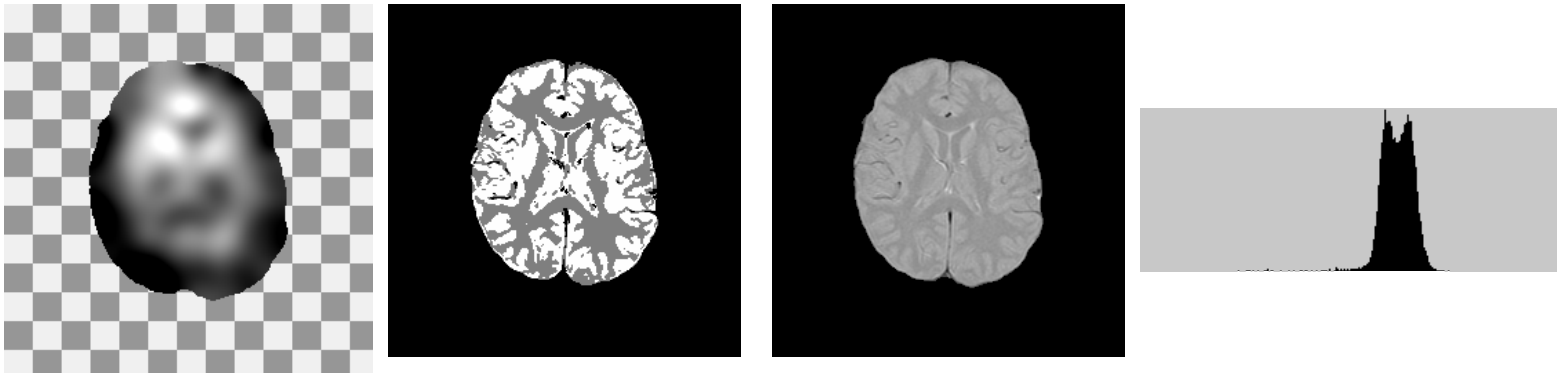
Zhang, Smith, Brady, IEEE Trans. Med. Im. 20(1), 45, 2001

Experiments on Brain MR Images

MAP analysis



MRF analysis



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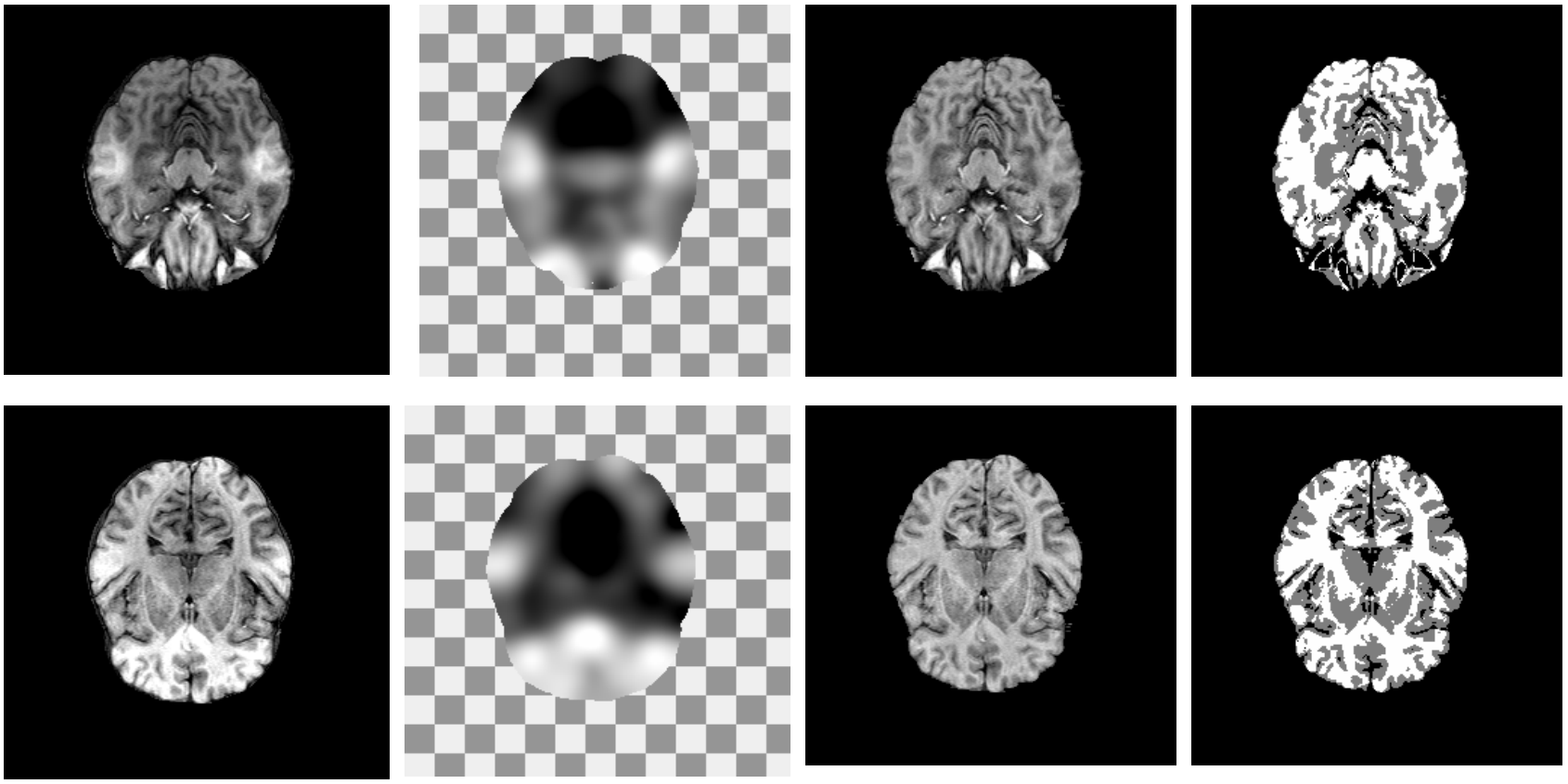
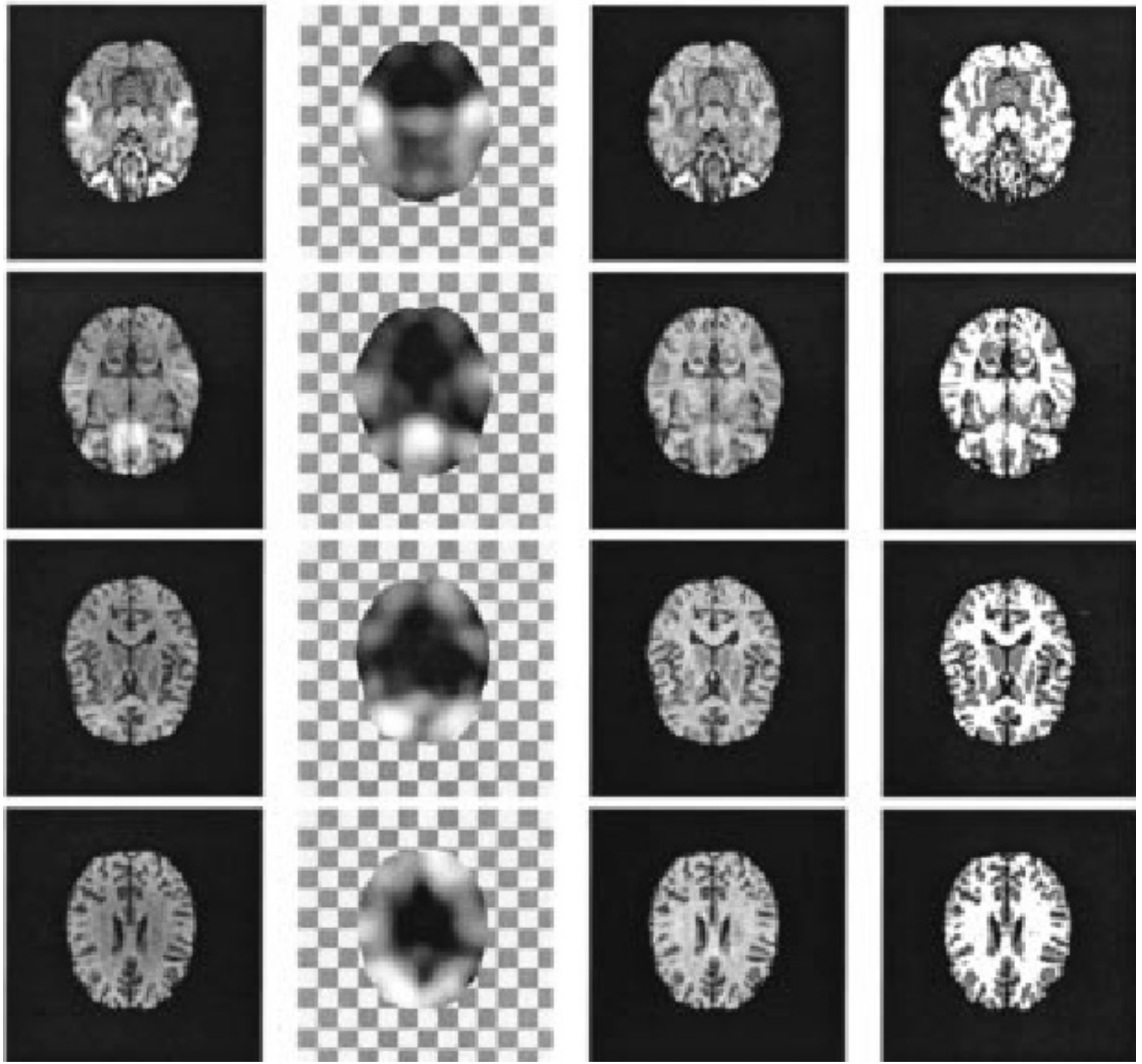
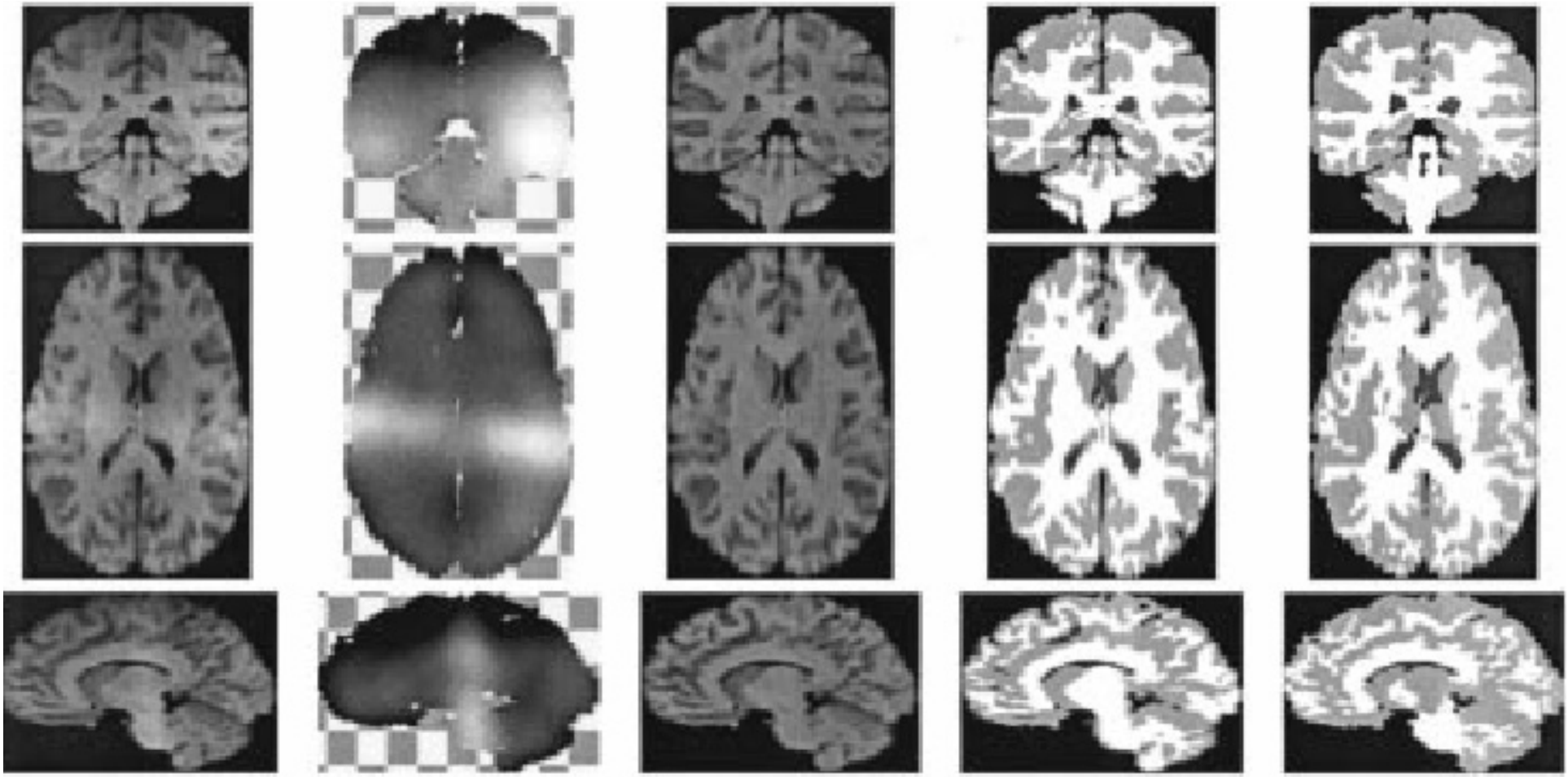
Experiments on Brain MR Images





L-to-R: original image; estimated bias field; corrected image; and segmentation



automatic HMRF segmentation

manual segmentation by a skilled clinician