Interval Estimation & Hypothesis Test

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March 12, 2025

What is Interval Estimation?

- **Interval estimation** is a range of plausible values for a parameter (e.g., a mean or a probability) constructed from observed data.
- In contrast to a *point estimate*, which gives a single value, an *interval* provides a measure of uncertainty or confidence.
- Often reported as **confidence intervals** (CIs).

Confidence Interval: Basic Concept

- A **confidence interval** for a parameter θ is given by random interval $[\mathsf{L}(\mathbf{X}), \mathsf{U}(\mathbf{X})]$, where $\mathbf{X}$ represents the sample data.
- In the case of a $(1-\alpha) \times 100\%$ confidence interval, we have:

$$\mathsf{Pr}\big(\mathsf{L}(\mathbf{X}) \leq \theta \leq \mathsf{U}(\mathbf{X})\big) = 1 - \alpha,$$

  for repeated sampling from the same population.
- Commonly, $\alpha = 0.05$ for a 95% confidence interval.

Types of Confidence Intervals

- **For Mean (when population standard deviation is known)**:

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

- **For Mean (when population standard deviation is unknown)**:

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

  where t^* is the critical value from the t-distribution.

- **For Proportion**:

$$\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Where:

- $\hat{p}$ is the sample proportion.
- z^* is the critical value from the standard normal distribution.

Example: Confidence Interval for a Mean (Normal Case)

Assumptions:

- $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ is a sample from a Normal distribution $N(\mu, \sigma^2)$.
- σ^2 is known, for simplicity (Z-interval).

Example: Confidence Interval for a Mean (Normal Case)

Steps to build a $(1-\alpha)\times 100\%$ CI for μ:

1. Compute sample mean: $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
2. Use standard error SE $= \frac{\sigma}{\sqrt{n}}$.
3. The $(1-\alpha)\times 100\%$ **Z-interval** is:

$$\left[\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right],$$

where $z_{\alpha/2}$ is the critical value from the standard normal distribution (e.g., $z_{0.025} \approx 1.96$ for a 95% CI).

Example

Suppose you conduct a survey and find the average height of 100 individuals to be 170 cm with a sample standard deviation of 10 cm. You want to estimate the average height of the population with a 95% confidence level.

Example

Suppose you conduct a survey and find the average height of 100 individuals to be 170 cm with a sample standard deviation of 10 cm. You want to estimate the average height of the population with a 95% confidence level.

$$\bar{x} = 170, \quad s = 10, \quad n = 100, \quad \text{Confidence Level} = 95\%$$

Since the population standard deviation is unknown, use the t-distribution:

$$SE = \frac{10}{\sqrt{100}} = 1$$

With a 95% confidence level and 99 degrees of freedom ($n - 1$), the t-value is approximately 1.984.

$$CI = 170 \pm 1.984 \cdot 1 = 170 \pm 1.984$$

$$CI = [168.016, 171.984]$$

Interpretation of the Confidence Interval

- If we repeat the experiment many times:
  - A certain percentage (e.g., 95%) of the intervals computed will contain the true mean μ.
- *Important*: The parameter μ is *fixed*; it's the interval that varies from sample to sample.
- Similar ideas extend to more complex scenarios (unknown σ^2, or building intervals for other parameters like proportions, difference of means, regression coefficients, etc.).

Choosing Between the Z-Score and T-Distribution

- Use a **z-score** when the sample size is large ($n \geq 30$) and/or when the population standard deviation (σ) is known.
- Use a **t-distribution** when the sample size is small ($n < 30$) and the population standard deviation (σ) is unknown. The t-distribution accounts for the additional uncertainty when estimating with smaller samples.

Pivotal Quantity

A **pivotal quantity** is a crucial concept in statistical inference, particularly for constructing confidence intervals and hypothesis testing. It's a function of the sample data and the unknown parameter(s) that has a known probability distribution independent of those parameters. In other words, it's a statistic that doesn't depend on the value of the parameter being estimated, which makes it very useful for inference.

Mathematical Function

A pivotal quantity (Q) for a parameter θ is a function of the sample data $X_1, X_2, \ldots, X_n$ and the parameter θ such that the distribution of Q is known and does not depend on θ.
Mathematically:

$$Q(X_1, X_2, \ldots, X_n; \theta)$$

The distribution of Q is the same regardless of the true value of θ.

The primary use of a pivotal quantity is to construct confidence intervals and hypothesis tests because its distribution is known and fixed, enabling exact calculations without relying on approximate methods.

Examples of Pivotal Quantities

Normal Distribution with Known Variance

Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed (i.i.d.) random variables from a normal distribution: $X_i \sim N(\mu, \sigma^2)$, with σ^2 known. The pivotal quantity for estimating the mean μ is:

$$Q = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

Q follows a standard normal distribution $N(0, 1)$, which does not depend on the unknown mean μ. This pivotal quantity can be rearranged to construct a confidence interval for μ.

Examples of Pivotal Quantities

Normal Distribution with Unknown Variance

If $X_i \sim N(\mu, \sigma^2)$ with σ^2 unknown:

$$Q = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

This follows a t-distribution with $n - 1$ degrees of freedom, independent of μ and σ^2.

Hypothesis Test

Introduction

- Hypothesis testing is a fundamental statistical procedure that allows researchers to make inferences about population parameters based on sample data.
- The process involves formulating a specific hypothesis about a population parameter and then determining whether the observed sample data provides sufficient evidence to reject or fail to reject that hypothesis.

Role of Hypothesis Testing in Statistical Inference

Statistical inference is the process of drawing conclusions about populations based on samples. Hypothesis testing is one of the two main branches of statistical inference (the other being estimation). While estimation focuses on determining the likely value of a population parameter, hypothesis testing examines specific claims about that parameter.

The essential role of hypothesis testing is to provide a systematic, objective framework for:

- Making decisions based on data rather than subjective impressions
- Quantifying the strength of evidence against a claim
- Controlling the probability of making incorrect conclusions
- Standardizing the process of scientific inquiry

Hypothesis

Imagine you have a question about the world, like "Does this medicine actually work?" or "Is this coin fair?" A hypothesis test helps you answer that question using data.

How Hypothesis Testing Works

- **Null hypothesis (H_0):** The boring, default assumption ("The medicine doesn't work" or "The coin is fair")
- **Alternative hypothesis (H_1):** What you're wondering might be true ("The medicine works" or "The coin is rigged")

Testing

- **Collect data:** Give the medicine to some people, flip the coin many times, etc.
- **Calculate a test statistic:** Turn your data into a single number that measures how far your results are from what you'd expect if the null hypothesis were true
- **Make a decision:** If your result is too unusual (assuming the null is true), you reject the null hypothesis in favor of the alternative

Example: Coin Flip

- Suppose H_o: "The coin is fair" (50% heads, 50% tails).
- You flip the coin 100 times and get 90 heads.
- If the coin were truly fair, getting 90 heads would be extremely unlikely.
- Since such an extreme result is rare if H_o were true, we reject H_o and suspect the coin is biased (H_1).

Fundamental Concepts in Hypothesis Testing

The foundation of hypothesis testing involves two competing claims about a population parameter:

Null Hypothesis (H_0)

The null hypothesis is formulated to be tested directly. By convention, it is written in terms of equality:

$$H_0 : \theta = \theta_0 \tag{1}$$

where θ represents the population parameter and θ_0 is a specific value.

Fundamental Concepts in Hypothesis Testing

Alternative Hypothesis (H_a or H_1)

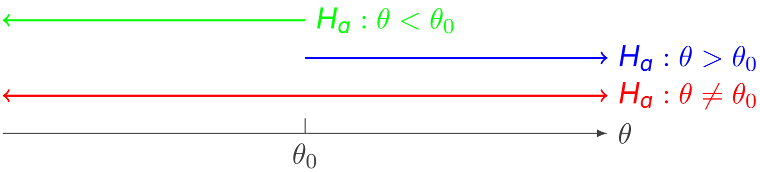
The alternative hypothesis is the claim accepted when the null hypothesis is rejected. It can be formulated in three ways:

$$H_a : \theta \neq \theta_0 \quad \text{(two-sided/two-tailed)} \tag{2}$$

$$H_a : \theta > \theta_0 \quad \text{(right-sided/right-tailed)} \tag{3}$$

$$H_a : \theta < \theta_0 \quad \text{(left-sided/left-tailed)} \tag{4}$$

Visual representation of different types of alternative hypotheses



Test Statistics

A test statistic is a numerical summary of the sample data that is used to make the decision about the null hypothesis. The choice of test statistic depends on:

- The parameter being tested
- The assumed distribution of the population
- The sample size

Z-statistic

Used when testing a mean with known population standard deviation or for large samples:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \tag{5}$$

where $\bar{X}$ is the sample mean, μ_0 is the hypothesized population mean, σ is the population standard deviation, and n is the sample size.

T-statistic

Used when testing a mean with unknown population standard deviation, especially for small samples:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \tag{6}$$

where s is the sample standard deviation.

Tests for a Single Mean

Z-test for a Mean (Known σ)

Assumptions:

- The population standard deviation σ is known
- Either the population follows a normal distribution, or the sample size is large enough for the Central Limit Theorem to apply

Hypotheses:

$$H_0 : \mu = \mu_0 \tag{7}$$

$$H_a : \mu \neq \mu_0 \text{ (or } \mu > \mu_0, \text{ or } \mu < \mu_0) \tag{8}$$

Test Statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \tag{9}$$

Decision Rule:

For a two-tailed test with significance level α:

- Reject H_0 if $|Z| > Z_{\alpha/2}$
- Fail to reject H_0 if $|Z| \leq Z_{\alpha/2}$

Example: Testing Student IQ Scores

Scenario:

- A coaching institute claims their students have a mean IQ score greater than 82 marks
- Population standard deviation is known: $\sigma = 20$
- Sample of 81 students has a mean score of 90 marks
- Confidence level: 95

Step 1: Define Hypotheses

$H_0 : \mu \leq 82$ (Mean IQ score is less than or equal to 82)

$H_1 : \mu > 82$ (Mean IQ score is greater than 82)

Test Type: Right-tailed test

Step 2: Set Significance Level

- Significance level: $\alpha = 0.05$
- For a right-tailed test with $\alpha = 0.05$:
  - Critical Z-value = 1.645

Step 3: Calculate Test Statistic

$$
\begin{aligned}
z &= \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \\
&= \frac{90 - 82}{20/\sqrt{81}} \\
&= \frac{8}{20/9} \\
&= \frac{8}{2.22} \\
&= 3.60
\end{aligned}
$$

Step 4: Make a Decision

- Calculated Z-value: 3.60
- Critical Z-value: 1.645
- Since $3.60 > 1.645$, we **reject** the null hypothesis

Step 5: Interpret Results

- There is sufficient statistical evidence at the 5% significance level to support the claim that the mean IQ score of students at this coaching institute is greater than 82 marks.

T-test for a Mean (Unknown σ)

Assumptions:

- The population standard deviation σ is unknown
- The population follows a normal distribution, or the sample size is large enough for the Central Limit Theorem to apply

Hypotheses:

$$H_0 : \mu = \mu_0 \tag{10}$$

$$H_a : \mu \neq \mu_0 \text{ (or } \mu > \mu_0, \text{ or } \mu < \mu_0) \tag{11}$$

Test Statistic:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \tag{12}$$

Decision Rule:

For a two-tailed test with significance level α and $n-1$ degrees of freedom:

- Reject H_0 if $|t| > t_{\alpha/2,n-1}$
- Fail to reject H_0 if $|t| \leq t_{\alpha/2,n-1}$

Example: Testing Student Exam Scores

Scenario:

- A professor claims that the average score on an exam is at least 75 points
- Population standard deviation is unknown
- Random sample of 16 students has a mean score of 71.5 points
- Sample standard deviation is 8.5 points
- Confidence level: 95

Step 1: Define Hypotheses

$$H_0 : \mu \geq 75 \quad \text{(Mean score is at least 75 points)} \tag{13}$$

$$H_1 : \mu < 75 \quad \text{(Mean score is less than 75 points)} \tag{14}$$

Test Type: Left-tailed test

Step 2: Set Significance Level

- Significance level: $\alpha = 0.05$
- Degrees of freedom: $df = n - 1 = 16 - 1 = 15$
- For a left-tailed test with $\alpha = 0.05$ and $df = 15$:
  - Critical t-value = -1.753

Step 3: Calculate Test Statistic

$$
\begin{aligned}
t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} && (15)\\
&= \frac{71.5 - 75}{8.5/\sqrt{16}} && (16)\\
&= \frac{-3.5}{8.5/4} && (17)\\
&= \frac{-3.5}{2.125} && (18)\\
&= -1.647 && (19)
\end{aligned}
$$

Step 4: Make a Decision

- Calculated t-value: -1.647
- Critical t-value: -1.753
- Since $-1.647 > -1.753$, we **fail to reject** the null hypothesis

Step 5: Interpret Results

- There is insufficient statistical evidence at the 5% significance level to conclude that the mean exam score is less than 75 points.
- Although the sample mean (71.5) is less than the claimed value (75), the difference is not statistically significant.

Introduction to Z-test for Two Means

- The Z-test for two independent means is used when:
  - Comparing means from two independent populations
  - Both population standard deviations (σ_1 and σ_2) are known
  - Sample sizes are relatively large
  - Data from both populations follow approximately normal distributions
- **Hypotheses:**

$$H_0 : \mu_1 - \mu_2 = \Delta_0 \text{ (often } \Delta_0 = 0) \tag{20}$$

$$H_a : \mu_1 - \mu_2 \neq \Delta_0 \text{ (or } \mu_1 - \mu_2 > \Delta_0, \text{ or } \mu_1 - \mu_2 < \Delta_0) \tag{21}$$

Introduction to Z-test for Two Means

Test Statistic:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{22}$$

Decision Rule: For a two-tailed test with significance level α:

- Reject H_0 if $|Z| > Z_{\alpha/2}$
- Fail to reject H_0 if $|Z| \leq Z_{\alpha/2}$

Example: Comparing Two Teaching Methods

Scenario:

- A school district wants to compare two teaching methods for mathematics
- Method A: Traditional approach
- Method B: Interactive approach
- Null hypothesis: Both methods are equally effective
- Alternative hypothesis: Method B is more effective than Method A

Example Data

Sample Information:

- Method A (Population 1):
  - Sample size: $n_1 = 45$ students
  - Sample mean: $\bar{x}_1 = 72$ points
  - Known population standard deviation: $\sigma_1 = 12$ points
- Method B (Population 2):
  - Sample size: $n_2 = 50$ students
  - Sample mean: $\bar{x}_2 = 78$ points
  - Known population standard deviation: $\sigma_2 = 15$ points
- Significance level: $\alpha = 0.05$

Step 1: Define Hypotheses

$$H_0 : \mu_1 \geq \mu_2 \quad \text{(Method A is at least as effective as Method B)} \tag{23}$$
$$H_1 : \mu_1 < \mu_2 \quad \text{(Method B is more effective than Method A)} \tag{24}$$

text **Test Type:** Left-tailed test (because we're testing if $\mu_1 < \mu_2$)

Step 2: Set Significance Level

- Significance level: $\alpha = 0.05$
- For a left-tailed test with $\alpha = 0.05$:
  - Critical Z-value = -1.645

Step 3: Calculate Test Statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{25}$$

$$= \frac{72 - 78}{\sqrt{\frac{12^2}{45} + \frac{15^2}{50}}} \tag{26}$$

$$= \frac{-6}{\sqrt{\frac{144}{45} + \frac{225}{50}}} \tag{27}$$

$$= \frac{-6}{\sqrt{3.2 + 4.5}} \tag{28}$$

$$= \frac{-6}{\sqrt{7.7}} \tag{29}$$

$$= \frac{-6}{2.77} \qquad = -2.17 \tag{30}$$

Step 4: Make a Decision

- Calculated Z-value: -2.17
- Critical Z-value: -1.645
- Since $-2.17 < -1.645$, we **reject** the null hypothesis

Step 5: Interpret Results

- There is sufficient statistical evidence at the 5% significance level to conclude that Method B is more effective than Method A.
- The p-value for this test is approximately 0.015, which is less than our significance level of 0.05.
- The mean score for Method B (78) is significantly higher than the mean score for Method A (72).

Introduction to T-test for Two Means

- The t-test for two independent means is used when:
  - Comparing means from two independent populations
  - Population standard deviations are unknown
  - We assume equal population variances ($\sigma_1^2 = \sigma_2^2$)
  - Data from both populations follow approximately normal distributions
- Formula for the test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{31}$$

where s_p^2 is the pooled variance

Pooled Variance

- When we assume equal population variances, we combine (pool) the sample variances
- Formula for pooled variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \tag{32}$$

- Degrees of freedom: $df = n_1 + n_2 - 2$

Example: Comparing Study Methods

Scenario:

- A researcher wants to compare two study methods for a statistics course
- Method 1: Traditional textbook study
- Method 2: Interactive online modules
- Null hypothesis: Both methods produce equal test scores
- Alternative hypothesis: The methods produce different test scores

Example Data

Sample Information:

- Method 1 (Traditional):
  - Sample size: $n_1 = 12$ students
  - Sample mean: $\bar{x}_1 = 76$ points
  - Sample standard deviation: $s_1 = 8$ points
- Method 2 (Interactive):
  - Sample size: $n_2 = 15$ students
  - Sample mean: $\bar{x}_2 = 82$ points
  - Sample standard deviation: $s_2 = 7$ points
- Significance level: $\alpha = 0.05$

Step 1: Define Hypotheses

$$H_0 : \mu_1 = \mu_2 \quad \text{(Both study methods produce equal test scores)} \tag{33}$$
$$H_1 : \mu_1 \neq \mu_2 \quad \text{(The study methods produce different test scores)} \tag{34}$$

text **Test Type:** Two-tailed test

Step 2: Set Significance Level

- Significance level: $\alpha = 0.05$
- Degrees of freedom: $df = n_1 + n_2 - 2 = 12 + 15 - 2 = 25$
- For a two-tailed test with $\alpha = 0.05$ and $df = 25$:
  - Critical t-values = ± 2.060

Step 3: Calculate Pooled Variance

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \tag{35}$$

$$= \frac{(12 - 1)(8)^2 + (15 - 1)(7)^2}{12 + 15 - 2} \tag{36}$$

$$= \frac{11 \cdot 64 + 14 \cdot 49}{25} \tag{37}$$

$$= \frac{704 + 686}{25} \tag{38}$$

$$= \frac{1390}{25} \qquad = 55.6 \tag{39}$$

text Pooled standard deviation: $s_p = \sqrt{55.6} = 7.46$

Step 4: Calculate Test Statistic

$$
\begin{aligned}
t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} &= \frac{76-82}{\sqrt{55.6\left(\frac{1}{12}+\frac{1}{15}\right)}} && (40)\\
= \frac{-6}{\sqrt{55.6\left(\frac{15+12}{12\cdot 15}\right)}} &= \frac{-6}{\sqrt{55.6\left(\frac{27}{180}\right)}} && (41)\\
= \frac{-6}{\sqrt{55.6\cdot 0.15}} &= \frac{-6}{\sqrt{8.34}} && (42)\\
= \frac{-6}{2.89} &= -2.08 && (43)
\end{aligned}
$$

Step 5: Make a Decision

- Calculated t-value: -2.08
- Critical t-values: ± 2.060
- Since $|-2.08| > 2.060$, we **reject** the null hypothesis

Step 6: Interpret Results

- There is sufficient statistical evidence at the 5% significance level to conclude that the two study methods produce different test scores.
- The interactive online modules (Method 2) produced significantly higher test scores (mean = 82) than the traditional textbook method (mean = 76).

Welch's T-test for Two Independent Means (Unknown and Unequal σ)

Assumptions:

- The population standard deviations are unknown and possibly unequal
- The samples are independent
- The populations follow normal distributions, or the sample sizes are large enough for the Central Limit Theorem to apply

Hypotheses:

$$H_0 : \mu_1 - \mu_2 = \Delta_0 \text{ (often } \Delta_0 = 0) \tag{44}$$

$$H_a : \mu_1 - \mu_2 \neq \Delta_0 \text{ (or } \mu_1 - \mu_2 > \Delta_0, \text{ or } \mu_1 - \mu_2 < \Delta_0) \tag{45}$$

Welch's T-test for Two Independent Means (Unknown and Unequal σ)

Test Statistic:

$$t' = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{46}$$

Degrees of Freedom: The degrees of freedom for Welch's t-test are approximated using the Welch-Satterthwaite equation:

$$df \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \tag{47}$$

Welch's T-test for Two Independent Means (Unknown and Unequal σ)

Decision Rule: For a two-tailed test with significance level α:

- Reject H_0 if $|t'| > t_{\alpha/2,df}$
- Fail to reject H_0 if $|t'| \leq t_{\alpha/2,df}$

Study Overview

Scenario:

- A pharmaceutical company is testing a new drug against an existing drug
- The primary outcome measure is patient recovery time (in days)
- Two independent groups of patients were studied
- Research question: Does the new drug improve recovery time?

Data Collection

Recovery Times (in days):

New Drug	12, 14, 11, 15, 13, 10, 9, 16, 12, 14
Old Drug	18, 20, 22, 19, 21, 25, 23, 17, 20, 22

Sample Size: 10 patients per group

Hypotheses

$$H_0 : \mu_{\text{new}} \geq \mu_{\text{old}} \quad \text{(New drug is not better)} \tag{48}$$

$$H_1 : \mu_{\text{new}} < \mu_{\text{old}} \quad \text{(New drug reduces recovery time)} \tag{49}$$

Test Type: One-tailed test (left-tailed)

Significance Level: $\alpha = 0.05$

Descriptive Statistics

Group	Sample Size	Mean (days)	Standard Deviation
New Drug	10	12.6	2.22
Old Drug	10	20.7	2.41

- Mean difference: 8.1 days in favor of the new drug
- Similar variability in both groups

Statistical Analysis

Welch's t-test for two independent means:

$$t = \frac{\bar{x}_{\text{new}} - \bar{x}_{\text{old}}}{\sqrt{\frac{s^2_{\text{new}}}{n_{\text{new}}} + \frac{s^2_{\text{old}}}{n_{\text{old}}}}} \tag{50}$$

$$= \frac{12.6 - 20.7}{\sqrt{\frac{2.22^2}{10} + \frac{2.41^2}{10}}} \qquad = \frac{-8.1}{\sqrt{\frac{4.93}{10} + \frac{5.81}{10}}} \tag{51}$$

$$= \frac{-8.1}{\sqrt{0.493 + 0.581}} \qquad = \frac{-8.1}{\sqrt{1.074}} \tag{52}$$

$$= \frac{-8.1}{1.036} \qquad = -7.82 \tag{53}$$

Degrees of Freedom Calculation

Welch-Satterthwaite equation:

$$df = \frac{\left(\frac{s_{\text{new}}^2}{n_{\text{new}}} + \frac{s_{\text{old}}^2}{n_{\text{old}}}\right)^2}{\frac{\left(\frac{s_{\text{new}}^2}{n_{\text{new}}}\right)^2}{n_{\text{new}}-1} + \frac{\left(\frac{s_{\text{old}}^2}{n_{\text{old}}}\right)^2}{n_{\text{old}}-1}} \tag{54}$$

$$= \frac{\left(\frac{2.22^2}{10} + \frac{2.41^2}{10}\right)^2}{\frac{\left(\frac{2.22^2}{10}\right)^2}{9} + \frac{\left(\frac{2.41^2}{10}\right)^2}{9}} \tag{55}$$

$$\approx 17.89 \tag{56}$$

Rounded: $df = 17$

Results

- Test statistic: $t = -7.82$
- Degrees of freedom: $df = 17.89$
- Critical t-value (one-tailed, $\alpha = 0.05$): $t_{\text{critical}} = -1.74$
- p-value: $p = 1.76 \times 10^{-7}$

Statistical Decision

- Since $t = -7.82 < t_{\text{critical}} = -1.74$, we **reject** the null hypothesis
- There is strong statistical evidence that the new drug reduces recovery time

Clinical Significance

- The new drug reduces mean recovery time by 8.1 days
- This represents a 39% reduction in recovery time
- 95% Confidence Interval for the difference: [5.9, 10.3] days
- The reduction is both statistically significant and clinically meaningful

Interval Estimation & Hypothesis Test

Thank You for Listening!